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Canonical Entropy of Reissner-Nordstrom Black Hole

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Recently, Hawking radiation of the black hole has been studied using the tunnel effect method. It is found the radiation spectrum of the black hole is not a strictly pure thermal spectrum. How the departure from pure thermal spectrum affects the entropy? This is a very interesting problem. In this paper, we calculate the partition function by energy spectrum obtained by tunnel effect. Using the relation between the partition function and entropy, we derive the expression of entropy the general charged black hole. In our calculation, we not only consider the correction to the black hole entropy due to fluctuation of energy but also consider the effect of the change of the black hole charges on entropy. We discuss Reissner-Nordstrom black hole and obtain that Reissner-Nordstrom black hole cannot approach the extreme black hole by changing its charges.

KEY WORDS: tunnel effect; canonical entropy; extreme black hole.

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1. INTRODUCTION

Hawking (1975) interpreted the quantum effect of the black hole as emitting thermal radiant spectrum from event horizon, which set a milestone in black hole physics. The discovery of this effect not only solved the problem in black hole thermodynamics but also announced the relation among quantum mechanics, thermodynamics and gravitation. Discovering the thermal properties of various black holes is an important subject of black hole physics. Hawking pointed that vacuum fluctuation near the surface of the black hole would produce virtual particle pair. When the virtual particles with negative energy come into black hole via tunnel effect, the energy of the black hole will decrease. At the same time, the particle with positive energy may thread out the

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gravitation region outside the black hole. Equivalently, the black hole radiates a particle. However, Hawking testifies that there is not any potential barrier in the tunnel.

Parikh and Wilczek (2000) discussed Hawking radiation by tunnel effect. They thought that tunnels in the process of the particle radiation had no potential barrier before particles radiated. Potential barrier is produced by radiation particles itself. That is, during the process of tunnel effect creation, the energy of the black hole decreases and the radius of the black hole horizon reduces. The horizon radius becomes a new value that is smaller than the original value. The decrease of radius is determined by the value of energy of radiation particles. There is a classical forbidden band– potential barrier between original radius and the one after the black hole radiates. Parikh and Wilczek skillfully obtained the radiation spectrum of Schwarzschild and Reissner-Nordstrom black holes. Angheben *et al.* (2005); Liu *et al.* (2006); Medved (2002a,b); Parikh (2002, 2004); Vagenas (2001, 2002a,b); Vagenas (2003); Zhang and Zhao (2005a,b,c) developed the method proposed by Parikh and Wilczek. They derived the radiation spectrum of the black hole in all kinds of space-time. Arzano *et al.* (2005, 2006); Medved and Vagenas (2005a,b) obtained radiation spectrum of Hawking radiation after considering the generalized uncertainty relation. And Angheben, Nadalini, Vanzo and Zerbini have computed the radiation spectrum of the arbitrary dimensional black hole and obtained the energy spectrum of radiation particles of general black hole (Angheben *et al.*, 2005; Zhao *et al.*, 2006).

Recently, to discuss the logarithm correction to the black hole entropy, various methods have been proposed (Camellia *et al.*, 2004a,b; Cavaglia *et al.*, 2003, 2004; Chatterjee and Majumdar, 2004, 2005; Chen, 2003; Kaul and Majumdar, 2000; More, 2005; Medved and Vagenas, 2004; Setare, 2004a,b; Setare, 2006). However, the effect of the change of the black hole charges on entropy and the correction to entropy due to the departure of radiation spectrum from pure thermal spectrum have not been discussed. In this paper, first we obtain the radiation spectrum of the black hole by quantum statistical method. In Section 3, using this radiation spectrum, we calculate the black hole entropy and derive the entropy of the charged black hole. In Section 4, we discuss Reissner-Nordstrom black hole and obtain the condition that the energy and charges satisfy when the logarithmic term is not divergent. The conclusion is given in Section 5. We take the simple function form of temperature ($c = \hbar = G = K_B = 1$).

2. RADIATION SPECTRUM OF THE CANONICAL CHARGED BLACK HOLE

For static and stationary space-times, since the metric is not related to time variable, in the spatial region we can constitute a contemporaneous plane that surrounds the black hole. We make the black hole dip in a thermal radiant field with temperature $T(T$ is the radiation temperature of the black hole). We make $R \gg r_H$, where R is the radius location of the contemporaneous plane, r_H is the horizon location of the black hole. Since the radius of the contemporaneous plane is very larger than the horizon radius of the black hole, we can take the region surrounded by this contemporaneous plane as an isolated thermodynamic system with conservation of energy. This region can be divided into three parts: the naked black hole, horizon surface and radiation field. Suppose that the total energy is E^0, the total charges is Q^0, the initial energy of the naked black hole is E, the charge is Q. E is Arnowitt-Deser-Misner (ADM) mass. Initial energy of the horizon surface of the black hole and the charge are zero respectively. The energy of the radiation field is E_r, the charge is Q_r. We know that the location of the black hole horizon r_H is a function with respect to energy and charge. It is denoted $r_H(E, Q)$. When the black hole have Hawking radiation, at first, the horizon location change. Suppose that the energy of Hawking radiation particles is E_s, and the electric charge is Q_n, the horizon location will change from $r_H(E, Q)$ to $r_H(E - E_s, Q - Q_n)$. At this time, there is a quantum energy layer with energy E_s and the electric charge is Q_n between the two horizons. Because the black hole and the horizon are dipped in a thermal radiant field with temperature T, the temperature is invariant during the creation process of radiation. Therefore, we can suppose that in this process the temperature of the black hole is invariant. This hypothesis is consistent with the one given in the case discussing Hawking radiation via tunnel effect. At this time, there are not energy commutation between energy layer and the radiation field. As a result, the energy of energy layer and the energy of naked black hole are conservative. And the charge of energy layer and the charge of naked black hole are conservative.

When the quantum energy layer is at state s with the charge Q_n and energy E_s, the naked black hole can be at any microscopic state with the charge $Q - Q_n$ and energy $E - E_s$. Let $\Omega(E - E_s, Q - Q_n)$ denote the number of microscopic state of the naked black hole with the charge $q = Q - Q_n$ and energy $E_b = E - E_s$. When the quantum energy layer is at state s, the number of microscopic state of the naked black hole and quantum energy layer is $\Omega(E - E_s, Q - Q_n)$. According to the principle of equal probability, every microscopic state of the compound system that consists of the naked black hole and quantum energy layer appears with the same probability. The probability that quantum energy layer is at state s is proportional to $\Omega(E - E_s, Q - Q_n)$. That is

$$\rho_{sn} \propto \Omega(E - E_s, Q - Q_n). \tag{1}$$

Because the number of microscopic state of the system is very big, for convenience, we discuss $\ln \Omega$. We expand $\ln \Omega$ as a power series with respect to E_s and Q_n. We have

$$\ln\Omega(E-E_s,Q-Q_n)=\ln\Omega(E,Q)+\left(\frac{\partial\ln\Omega}{\partial E_b}\right)_{E_s=0,Q_n=0}(-E_s)$$
$$+\frac{1}{2}\left(\frac{\partial^2\ln\Omega}{\partial E_b^2}\right)_{E_s=0,Q_n=0}E_s^2+\cdots+\left(\frac{\partial\ln\Omega}{\partial q}\right)_{E_s=0,Q_n=0}(-Q_n)$$
$$+\frac{1}{2}\left(\frac{\partial^2\ln\Omega}{\partial q^2}\right)_{E_s=0,Q_n=0}Q_n^2+\left(\frac{\partial^2\ln\Omega}{\partial q\partial E_b}\right)_{E_s=0,Q_n=0}E_sQ_n+\cdots. \quad (2)$$

The first term in the right side of Eq. (2) is a constant. So (1) can be rewritten as

$$\rho_{sn}\propto\exp\left[-\beta E_s+\beta_2E_s^2-\alpha Q_n+\alpha_2Q_n^2+\gamma_{11}E_sQ_n+\cdots\right], \quad (3)$$

where

$$\alpha_k=\frac{1}{k!}\left(\frac{\partial^k\ln\Omega}{\partial q^k}\right)_{E_s=0,Q_n=0},\quad \beta_k=\frac{1}{k!}\left(\frac{\partial^k\ln\Omega}{\partial E_b^k}\right)_{E_s=0,Q_n=0},$$
$$\gamma_{11}=\left(\frac{\partial^2\ln\Omega}{\partial E_b\partial q}\right)_{E_s=0,Q_n=0}.$$

According to the view of statistical physics, logarithm of the number of microscopic state of the system should be the entropy of the system. That is

$$S=\ln\Omega. \quad (4)$$

Then (2) can be expressed as

$$S(E-E_s,Q-Q_n)-S(E,Q)=-\alpha Q_n+\alpha_2Q_n^2+\cdots-\beta E_s+\beta_2E_s^2$$
$$+\gamma_{11}E_sQ_n+\cdots, \quad (5)$$

in (5), $S(E-E_s,Q-Q_n)-S(E,Q)$ is the difference between the entropy before the naked black hole radiates and the entropy after the naked black hole radiates. That is

$$\Delta S=S(E-E_s,Q-Q_n)-S(E,Q). \quad (6)$$

Based on Eq. (3), the energy spectrum of the black hole radiation is as follows:

$$\rho_{sn}\propto e^{\Delta S}. \quad (7)$$

where according to thermodynamics relation, β should be derivative of the temperature.

Normalizing the distribution function, we obtain

$$\rho_{sn}=\frac{1}{Z_G}e^{\Delta S}=\frac{1}{Z_G}e^{-\beta[E_s-\Phi Q_n]+\alpha_2Q_n^2+\cdots+\beta_2E_s^2+\gamma_{11}E_sQ_n+\cdots}, \quad (8)$$

where $\Phi = (\frac{\partial E_b}{\partial q})_{E_s=0, Q_n=0}$ is a static electric potential at the black hole horizon, Z_G is named as a canonical partition function. It is defined as

$$Z_G = \sum_{s,n} e^{\Delta S}, \tag{9}$$

If we adopt the semi-classical method, the partition function in Eq. (9) can be rewritten as

$$Z_G = \int dE_s dQ_n \rho(E - E_s, Q - Q_n) e^{-\beta[E_s - \Phi Q_n] + \alpha_2 Q_n^2 + \cdots + \beta_2 E_s^2 + \gamma_{11} E_s Q_n + \cdots}, \tag{10}$$

where $\rho(E - E_s, Q - Q_n)$ is the state density when the energy and the charge are respectively $E - E_s$, $Q - Q_n$. When the energy and the charge are invariable, the corresponding system is a microcanonical system. The relation between the density of the state and entropy is

$$\rho(E - E_s, Q - Q_n) = \exp\left[S_{MC}(E - E_s, Q - Q_n)\right]. \tag{11}$$

Provided the microcanonical entropy $S_{MC}(E - E_s, Q - Q_n)$ can be Taylor-expanded around the average equilibrium energy E and Q

$$\begin{aligned} S_{MC}(E - E_s, Q - Q_n) &= S_{MC}(E, Q) - \alpha Q_n + \alpha_2 Q_n^2 \\ &+ \cdots - \beta E_s + \beta_2 E_s^2 + \gamma_{11} E_s Q_n + \cdots, \end{aligned} \tag{12}$$

When we neglect the higher-order small term, the partition function (10) can be rewritten as

$$Z_G = \int dE_s dQ_n e^{S_{MC}(E,Q)} e^{2[-\beta E_s - \alpha Q_n + \alpha_2 Q_n^2 + \beta_2 E_s^2 + \gamma_{11} E_s Q_n/2]}, \tag{13}$$

When $\Phi < 0$, the integral with respect to E_s and Q_n have a contribution under the case that E_s and Q_n are very small. Let the integral upper limit be infinity. We have

$$\begin{aligned} Z_G(\beta, \alpha) &= e^{S_{MC}(E,Q)} \int dQ_n e^{2\left[-\alpha Q_n + \alpha_2 Q_n^2\right]} \left[\frac{1}{2}\sqrt{\frac{\pi}{-2\beta_2}}\right. \\ &\quad \left. \times \exp\left(\frac{(\beta - \gamma_{11} Q_n)^2}{-2\beta_2}\right)\left(1 - erf\left(\frac{\beta - \gamma_{11} Q_n}{\sqrt{-2\beta_2}}\right)\right)\right] \\ &\approx e^{S_{MC}(E,Q)} \left[\frac{1}{2}\sqrt{\frac{\pi}{-2\beta_2}} \exp\left(\frac{\beta^2}{-2\beta_2}\right)\left(1 - erf\left(\frac{\beta}{\sqrt{-2\beta_2}}\right)\right)\right] \\ &\quad \times \left[\frac{1}{2}\sqrt{\frac{\pi}{-2(\alpha_2 + 2\alpha\gamma_{11}\beta)}} \exp\left(\frac{\alpha^2}{-2(\alpha_2 + 2\alpha\gamma_{11}\beta)}\right)\right. \\ &\quad \left. \times \left(1 - erf\left(\frac{\alpha}{\sqrt{-2(\alpha_2 + 2\alpha\gamma_{11}\beta)}}\right)\right)\right]. \end{aligned} \tag{14}$$

where

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

is a error integral.

3. CANONICAL ENTROPY OF CHARGED BLACK HOLE

Using the standard formula from equilibrium statistical mechanics

$$S = \ln Z_G - \beta \frac{\partial \ln Z_G}{\partial \beta} - \alpha \frac{\partial \ln Z_G}{\partial \alpha}. \tag{15}$$

It is easy to deduce that the canonical entropy is given in terms of the microcanonical entropy by

$$S_C(E, Q) = S_{MC}(E, Q) + \Delta_S, \tag{16}$$

where

$$\Delta_S = \ln f(\beta, \beta_2) - \beta \frac{\partial \ln f(\beta, \beta_2)}{\partial \beta} + \ln f(\alpha, \alpha_2) - \alpha \frac{\partial \ln f(\alpha, \alpha_2)}{\partial \alpha}, \tag{17}$$

$$f(\beta, \beta_2) = \frac{1}{2}\sqrt{\frac{\pi}{-2\beta_2}} \exp\left(\frac{\beta^2}{-2\beta_2}\right)\left[1 - erf\left(\frac{\beta}{\sqrt{-2\beta_2}}\right)\right],$$

According to the asymptotic expression of the error function

$$erf(z) = 1 - \frac{e^{-z^2}}{\sqrt{\pi} z}\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{(2z^2)^k}\right], \quad |z| \to \infty,$$

we obtain

$$f(\beta, \beta_2) = \frac{1}{2\beta}\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{2^k}\left(\frac{\sqrt{-2\beta_2}}{\beta}\right)^{2k}\right]. \tag{18}$$

When $\Phi < 0$, we can derive the logarithmic correction term.

$$\Delta_S = \ln\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{2^k C^k}\right] + \ln\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{2^k}\left(\frac{\sqrt{\frac{\partial}{\partial Q}(\beta\Phi)(1 + 2\beta^2\Phi\frac{\partial Q}{\partial E})}}{-\beta\Phi}\right)^{2k}\right] + 2\ln T - \ln|\Phi|.$$

$$= \ln\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{2^k C^k}\right]$$
$$+ \ln\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{2^k} y^k (1 + 2\beta^2 \Phi \frac{\partial Q}{\partial E})^k\right] + 2\ln T - \ln|\Phi|\,. \tag{19}$$

where C is thermal capacity under the case that the charge is invariable

$$C \equiv -\beta^2 \left(\frac{\partial E}{\partial \beta}\right)_Q, \tag{20}$$

and

$$\beta_2 = -\frac{1}{2}\frac{\beta^2}{C}, \quad y = \left(\frac{\partial}{\partial Q}\left(\frac{1}{\beta \Phi}\right)\right)_E. \tag{21}$$

In error function, we take the sum from k equals one to n as the approximate value of the series. When z is a real number, its error does not exceed the absolute value of the first term that has been neglected in the series. Therefore, when $C < -1$ or $C > 1$, the first term in Δ_S is not divergent. However, when $-1 \le C \le 1$, the entropy may be divergent. The physics property implied by the fact that the black hole entropy is divergent need further discuss. In the second term of Δ_S, when $|y| < 1$ and $\left|2\beta^2\Phi\frac{\partial Q}{\partial E}\right| < 1$, it is not divergent. Therefore, we can obtain the relation between the energy and the charges.

4. APPLICATION EXAMPLE

The linear element of Reissner-Nordstrom black hole:

$$dS^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{22}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \tag{23}$$

The radiation temperature of the black hole

$$T = \frac{r_+ - r_-}{4\pi r_+^2}, \tag{24}$$

where $r_\pm$ are solutions of equation $f(r) = 0$. $r_+ = M + \sqrt{M^2 - Q^2}$ is radius of the black hole event horizon. The thermal capacity is as follows:

$$C = \left(\frac{\partial E}{\partial T}\right)_Q = \frac{2\pi r_+^2(r_+ - r_-)}{3r_- - r_+} = \frac{2\pi M^2(1 + \sqrt{1-x^2})^2\sqrt{1-x^2}}{1 - 2\sqrt{1-x^2}}. \tag{25}$$

where $Q^2 = x^2M^2$. From (25), when $0 < x^2 < \frac{3}{4}$, $M^2 > \frac{1}{8\pi}$, it can ensure $C < -1$. So the logarithmic term of the black hole is not divergent.

According to Jiang and Wu (2006), let $\Phi = -\frac{Q}{r_+}$, we have

$$y = \left(\frac{\partial}{\partial Q}\left(\frac{1}{\beta\Phi}\right)\right)_E = \frac{1 + (1 - x^2)^{3/2}}{2\pi M^2x^2\sqrt{1 - x^2}(1 + \sqrt{1 - x^2})^2}. \tag{26}$$

Based on $|y| < 1$, we obtain

$$1 + (1 - x^2)^{3/2} < 2\pi M^2x^2\sqrt{1 - x^2}(1 + \sqrt{1 - x^2})^2. \tag{27}$$

For non-extreme black hole, we have $0 < x^2 < 1$. When the higher-order small quantities are neglected, we can approximately derive that the logarithmic term of the black hole is not divergent, when

$$\frac{1}{4\pi M^2} < x^2 < 1. \tag{28}$$

Based on the above condition, $0 < x^2 < \frac{3}{4}$, $M^2 > \frac{4}{9\pi}$ and $\frac{1}{4\pi M^2} < x^2 < 1$, we derive that the logarithmic correction term of the black hole entropy is not divergent, when the energy and the charge of the black hole satisfies the following relation

$$M^2 > \frac{4}{9\pi}, \quad \frac{3}{4}M^2 > Q^2 > \frac{1}{4\pi}. \tag{29}$$

When $C \ll -1$, $y \ll 1$ and $r_+ \gg r_-$, from (19), we can obtain the canonical entropy is

$$S \approx S_{MC} + 2\ln T - \ln|\Phi| = S_{MC} - \ln S_{MC} - \ln|\Phi| + \text{const}. \tag{30}$$

When the energy of the black hole $M^2 < \frac{4}{9\pi}$, it cannot ensure the logarithmic term of the black hole entropy is not divergent. It means that the energy of charged Reissner-Nordstrom black hole has lower limit. However, from $\frac{3}{4}M^2 > Q^2 > \frac{1}{4\pi}$, we know that Reissner-Nordstrom does not approach the extreme black hole. This problem is open.

5. CONCLUSION

After considering the correction to the black hole thermodynamic quantities duo to thermal fluctuation, the expression of entropy is (Cavaglia and Fabbri, 2002; Gour and Medved, 2003; Setare, 2003, 2004a)

$$S = \ln\rho = S_{MC} - \frac{1}{2}\ln(CT^2) + \cdots, \tag{31}$$

There is a limitation in the above result. That is the thermal capacity of Schwarzschild black hole is negative. This leads to the entropy given by Eq. (31) is divergent. So this relation is not valid to Schwarzschild black hole. However, for general four-dimensional curved space-times, when we take a proper approximation or limit, they can return to Schwarzschild space-times. This implies that Eq. (31) has not universality. However, in our result we only request the thermal capacity satisfies $C < -1$ or $C > 1$. When the energy of Schwarzschild black hole satisfies $M^2 > 1/8\pi$, we obtain that the entropy is not divergent. It means that the energy of Schwarzschild black hole has lower limit.

From the demand to the thermal capacity of Reissner-Nordstrom black hole, we also can obtain that the energy of Reissner-Nordstrom black hole has lower limit. And based on the demand to y, we can derive the relation between the energy and the charge of the black hole. When we consider the demand of the thermal capacity and y, we obtain that Reissner-Nordstrom black hole cannot approach the extreme black hole by changing its charges.

In addition, the research of the black hole entropy is based on the fact that the black hole has thermal radiation and the radiation spectrum is a pure thermal spectrum. However, Hawking obtained that the radiation spectrum is a pure thermal spectrum only under the condition that the background of space-time is invariable. During this radiation process, there exist the information loss. The information loss of the black hole means that the pure quantum state will disintegrate to a mixed state. This violates the unitarity principle in quantum mechanics. When we discuss the black hole radiation by the tunnel effect method, after considering the conversation of energy and the change of the horizon, we derive that the radiation spectrum is no longer a strict pure thermal spectrum. This method can avoid the limit of Hawking radiation and point out that the self-gravitation provides the potential barrier of quantum tunnel.

Our discussion is based on the quantum tunnel effect of the black hole radiation. So our discussion is very reasonable. We provide a way for studying the quantum correction to Bekenstein-Hawking entropy. Based on our method, we can further check the string theory and single loop quantum gravitation theory and determine which one is perfect. We not only consider the correction to the black hole entropy due to fluctuation of energy but also consider the effect of the change of the black hole charges on entropy. For Reissner-Nordstrom black hole, when $M^2 < \frac{4}{9\pi}$, it cannot ensure the logarithmic term of the black hole entropy is not divergent. It means that the energy of Reissner-Nordstrom black hole has lower limit. However, the condition $\frac{3}{4}M^2 > Q^2 > \frac{1}{4\pi}$ implies that Reissner-Nordstrom black hole cannot approach the extreme one by changing its charges. The physics phenomenon behind this problem need further discuss.

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REFERENCES

Angheben, M., Nadalini, M., Vanzo, L., and Zerbini, S. (2005). **JHEP05**, 014.

Arzano, M. (2006). *Modern Physical Letters A* **21**, 41.

Arzano, M., Medved, A. J. M., and Vagenas, E. C. (2005). **JHEP09**, 037, [arXiv:hep-th/0505266]

Camellia, G. A., Arzano, M., and Procaccini, A. (2004a). *Physical Review D* **70**, 107501.

Camellia, G. A., Arzano, M., and Procaccini, A. (2004b). *Physical Review D* **70**, 107501.

Cavaglia, M. and Fabbri, A. (2002). *Physical Review D* **65**, 044012.

Cavaglia, M., Das, S., and Maartens, R. (2003). *Classical and Quantum Gravity* **20**, L205.

Cavaglia, M., Das, S., and Maartens, R. (2004). *Classical and Quantum Gravity* **21**, 4511.

Chatterjee, A. and Majumdar, P. (2004). *Physical Review Letters* **92**, 141301.

Chatterjee, A. and Majumdar, P. (2005). *Physical Review D* **71**, 024003.

Chen, P. (2003). Generalized Uncertainty Principle and dark matter, preprint Astro-ph/0305025.

Gour, G. and Medved, A. J. M. (2003). *Classical and Quantum Gravity* **20**, 3307.

Hawking, S. W. (1975). *Communications in Mathematical Physics* **43**, 199.

Hemming, S. and Keski-Vakkuri, (2001). *European Physical Review D* **64**, 044006, [arXiv:gr-qc/0005115].

Jiang, Q. Q. and Wu, S. Q. (2006). *Physics Letters B* **635**, 151.

Kaul, R. K. and Majumdar, P. (2000). *Physical Review Letters* **84**, 5255.

Liu, C. Z., Zhang, J. Y., and Zhao, Z. (2006). *Physics Letters B* **639**, 670.

Medved, A. J. (2002a). *Classical and Quantum Gravity* **19**, 589.

Medved, A. J. (2002b). *Physical Review D* **66**, 124009.

Medved, A. J. M. and Vagenas, E. C. (2004). *Physical Review D* **70**, 124021, [arXiv:hep-th/0411022].

Medved, A. J. M. and Vagenas, E. C. (2005a). *Modern Physical Letters A* **20**, 2449.

Medved, A. J. M. and Vagenas, E. C. (2005b). *Modern Physical Letters A* **20**, 1723, [arXiv:gr-qc/0505015].

More, S. S. (2005). *Classical and Quantum Gravity* **22**, 4129.

Parikh, M. K. (2002). *Physics Letters B* **546**, 189.

Parikh, M. K. (2004). Energy Conservation and Hawking Radiation, preprint arXiv:hep-th/0402166.

Parikh, M. K. and Wilczek, F. (2000). *Physical Review Letters* **85**, 5042.

Setare, M. R. (2003). *Physics Letters B* **573**, 173.

Setare, M. R. (2004a). *European Physical Journal C* **33**, 555.

Setare, M. R. (2004b). *Physical Review D* **70**, 087501.

Setare, M. R. (2006). *Internatinal Journal of Modern Physics A* **21**, 1325.

Vagenas, E. C. (2001). *Physics Letters B* **503**, 399.

Vagenas, E. C. (2002a). *Modern Physical Letters A* **17**, 609.

Vagenas, E. C. (2002b). *Physics Letters B* **533**, 302.

Vagenas, E. C. (2003). *Physics Letters B* **559**, 65.

Zhang, J. Y. and Zhao, Z. (2005a). *Physics Letters B* **618**, 14.

Zhang, J. Y. and Zhao, Z. (2005b). *Nuclear Physics B* **725**, 173.

Zhang, J. Y. and Zhao, Z. (2005c). *Nuclear Physics* **JHEP10**, 055.

Zhao, R., Zhang, L. C., and Hu, S. Q. (2006). *Acta Physica Sin.* **55**, 3898 (in Chinese);